ENTS 622: Introduction to Digital Communications Systems

1. Say \( Y(t) = X(t) + A \), where \( X(t) \) is a random process and \( A \) is a constant, known DC component. Then, please choose the best answer:

   (a) \( Y(t) \)'s autocorrelation function will contain a constant component equal to \( A \).
   
   (b) \( Y(t) \)'s autocorrelation function will contain a constant component equal to \( A^2 \).
   
   (c) \( Y(t) \)'s power spectral density will be shifted by \( A \) in the positive direction with respect to \( X(t) \)’s.
   
   (d) \( Y(t) \)'s power spectral density will contain a constant component equal to \( A\delta(f + A) \) (where \( \delta(\cdot) \) is the Dirac delta function).

2. Which is the Fourier transform of \( \text{rect}(\frac{t}{T}) = \begin{cases} 1, & |t| \leq \frac{T}{2} \\ 0, & \text{otherwise} \end{cases} \)

   (a) \( T \text{rect}(fT) \)
   
   (b) \( e^{-j2fT} \text{sinc}^2(f) \)
   
   (c) \( \frac{\text{rect}(f)}{2} (\delta(f - T) + \delta(f + T)) \)
   
   (d) \( T \text{sinc}(fT) \)

3. In the illustration below, \( x(t) \) is a real narrowband signal, with bandwidth \( 2W \) and carrier frequency \( f_c \gg W \). The low-pass filter (LPF) has bandwidth \( W \) and a gain of 1.

   \[ x(t) \xrightarrow{\cos(2\pi f_c t)} \text{LPF} \xrightarrow{2} y(t) \]

   Then \( y(t) \) is

   (a) the complex envelope of \( x(t) \).
   
   (b) the quadrature component of \( x(t) \).
   
   (c) the in-phase component of \( x(t) \).
   
   (d) the envelope of \( x(t) \).
4. Concerning DPSK (Differential Phase Shift Keying) and PSK (Phase Shift Keying) modulation, we may say that:

(a) The symbol error rate for DPSK is less than for PSK, for equal SNR.
(b) PSK is a constant envelope modulation method.
(c) DPSK uses non-coherent demodulation.
(d) Both (b) and (c).

5. The entropy of a discrete memoryless source

(a) is a lower bound on the average codeword length.
(b) is equal to the required bandwidth of a communication channel.
(c) is an upper bound on the SNR of a detector output.
(d) is zero if and only if none of the symbols’ probabilities are equal to 1.

6. If \( X = 48 \) in absolute linear scale, then approximately what is \( X \) decreased by 6dB, in absolute linear scale?

(a) 6
(b) 12
(c) 18
(d) 24

7. If we denote the natural logarithm as \( \ln (\cdot) \), and the Dirac delta function as \( \delta(\cdot) \), then please, choose which one is correct:

(a) \( \int_{-20}^{-1} \frac{3(x+3)}{e^{-x}} \, dx = \frac{3}{e^3} \)
(b) \( \int_{0}^{\infty} \delta(t + \frac{1}{2}) e^{-t^2} \, dt = \frac{\sqrt{\pi}}{2} \)
(c) \( \int_{-2}^{-1} (se^s + 1) \delta(s - \frac{1}{2}) \, ds = -\frac{1}{e^2} \)
(d) \( \int_{1}^{2} \delta(w + 1) \frac{3}{w^2} \, dw = -3 \cdot \ln(2) \)
8. Consider the signal defined over the signaling interval, \( T \), as:

\[ s(t) = \begin{cases} 
\frac{A}{2}, & \text{for } t \in [0, \frac{T}{2}] \\
-\frac{A}{2}, & \text{for } t \in [\frac{T}{2}, T] \\
0, & \text{otherwise}
\end{cases} \]

Please, select the impulse response for the matched filter of \( s(t) \).

9. If the impulse response of a root raised cosine filter is convolved with itself, we obtain a

(a) Dirac delta impulse response.
(b) rectangular impulse response.
(c) raised cosine impulse response.
(d) triangular impulse response.

10. If a random process is ergodic, it means that

(a) its power spectral density does not exist.
(b) its second order statistics are time-invariant.
(c) its expected value is zero.
(d) its statistical averages can be determined from its sample averages.